Statistical Analysis of Interference in Asynchronous MC-CDMA Systems

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Abstract

Two major sources of interference affect asynchronous MC-CDMA systems, i.e. multiple access interference due to subcarriers with the same frequency (MAI) and multiple access interference due to subcarriers with different frequency (ICI). Both MAI and ICI are generally modeled as zero-mean Gaussian random variable and their power has been previously been derived in the case of uniformly distributed timing offsets. In this paper, we derive an expression of the conditional power of the MAI and ICI as a function of timing offset. The advantage is that the interference power can then be derived for various distributions of the timing offsets. We then apply the expression to calculating the MAI and ICI power for two different distributions of timing offsets, i.e. uniform distribution and Poisson distribution. Finally, we propose a statistical model for asynchronous MC-CDMA systems that will simplify the computer simulation process of these systems. It is based on modeling the asynchronous system with a synchronous system followed by additive noise representing the MAI and ICI. The model is validated by comparing the BER at the output of the asynchronous system and the model.

1. Introduction

MC-CDMA systems [1] have recently attracted a lot of research in the world of wireless telecommunications. This is mainly due to the fact that MC-CDMA systems have several advantages over current DS-CDMA systems, such as good spectrum properties and robustness to frequency selective fading. However, under asynchronous transmission, the performance of MC-CDMA systems is significantly affected by multiple access interference (MAI) [2].

Asynchronous transmission refers to the scenario whereby multiple user signals arrive at the receiver with different delays which is also referred as timing offsets [3]. Asynchronous transmission is at the source of two types of interference in MC-CDMA systems. MAI is the interference created by other users transmitting information over the same sub-carrier frequencies which is also common to DS-CDMA systems. ICI is more specific to MC-CDMA systems and represents the interference generated by transmission over sub-carriers with different frequency. The sum of the MAI and ICI is a good approximation of the total interference affecting asynchronous MC-CDMA systems. One aim of this paper is to estimate the total interference power for different statistics of timing offsets.

Previous studies devoted to analysing interference in asynchronous MC-CDMA systems generally assume the timing offset 'τ' to be uniformly distributed over one symbol duration T_s [4]-[8]. In many applications, however, timing offsets are modelled as Poisson distributed [9]-[12]. In this paper, we derive an exact expression for the total interference power as a function of offset. The derived expression will allow the estimation of interference power for various distributions of timing offsets, including Poisson distribution. This result is then used to propose a computer simulation model for asynchronous MC-CDMA systems. Current models require heavy computational load. We propose to replace the asynchronous system with a synchronous system, which requires significantly less computing, followed by an additive noise component representing the MAI and ICI. The model is validated using computer simulations.

This paper is structured as follows. In section 2, we present the asynchronous MC-CDMA system considered in this paper. In section 3, we derive the total interference power affecting asynchronous MC-CDMA systems as a function of timing offsets. In Section 4, we calculate the interference power for two different distributions of the timing offsets, i.e. the uniform distribution and the Poisson distribution. The computer simulation model for asynchronous MC-CDMA systems is proposed in Section 5. Section 6 concludes this paper.

2. Asynchronous MC-CDMA Model

The asynchronous MC-CDMA model considered in this paper is shown in *Figure 1*. We consider K users transmitting data assumed i.i.d and BPSK modulated, then spread in the frequency domain. We also assume the total number of sub-carriers for each user is equal to the spreading factor.

The transmitted signal for the k^{th} user is given by

$$s_k(t) = \sum_{i=1}^{N} b_k(t) c_{k,i}(t) \cos(2\pi f_i t) \qquad (1)$$

where N is the spreading factor; $b_k(t)$ is the transmitted data bit ; $c_{k,i}(t)$ is the spreading code for the i^{th} subcarrier in the k^{th} user; f_i is the carrier frequency for the i^{th} subcarrier given by $f_i = f_c + \frac{i}{T_s}$ when $i = 1, 2\ldots N$ and f_c is the fundamental transmission frequency.

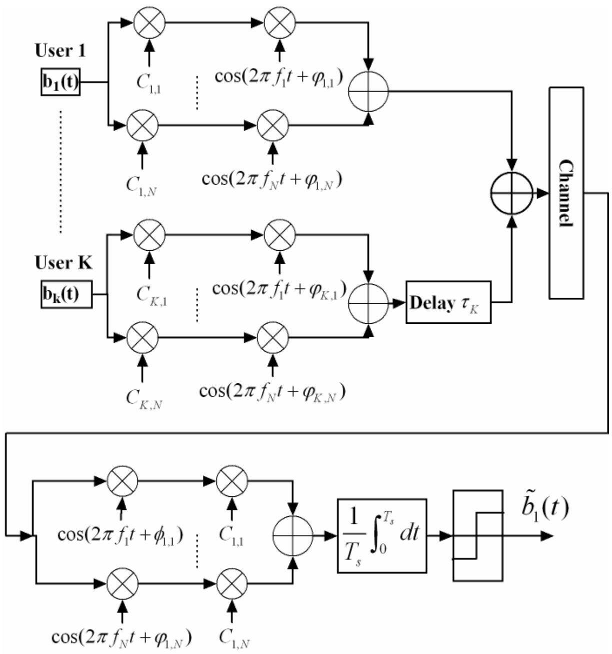

Figure 1. Asynchronous MC-CDMA model

In the considered asynchronous MC-CDMA model, it is assumed that the channel for each sub-carrier and each user is an independent frequency non-selective Rayleigh fading channel. To further simplify the problem we also assume that the statistics of the Rayleigh fading for all channels are the same. The complex impulse response of the channel for the i^{th} subcarrier in the k^{th} user can be written as

$$h_{k,i} = \alpha_{k,i} e^{j\psi_{k,i}} \delta(t - \tau_k) \qquad (2)$$

where $\alpha_{k,i}$ is a Rayleigh random variable with mean $E[\alpha_{k,i}] = \beta\sqrt{\frac{\pi}{2}}$, 2nd Moment $E[\alpha_{k,i}^2] = 2\beta^2$. β is the Rayleigh parameter. $\Psi_{k,i}$ is a random phase offset introduced by the channel and is modeled as uniformly distributed over the interval of $[0, 2\pi)$. τ_k represents the timing offset introduced by the k^{th} user.

Given the above channel characteristics, the received signal can be written as

$$r_k(t) = \sum_{i=1}^{N} \alpha_{k,i} b_k(t-\tau_k) c_{k,i}(t-\tau_k) \cdot \cos(2\pi f_i (t-\tau_k) + \psi_{k,i}) \qquad (3)$$

3. Interference analysis

3.1 Multiple access interference

In the rest of the paper, the reference user is referred to as user h. The MAI introduced by the k^{th} user over the i^{th} subcarrier path over one symbol duration is given by

$$I_{MAI,i,k} = \int_0^{T_s} \alpha_{k,i} b_k(t-\tau_k) \cdot c_{k,i}(t-\tau_k) c_{h,i}(t) \cos(2\pi f_i t) dt \cdot \cos\left[2\pi f_i (t-\tau_k) + \psi_{k,i}\right] \qquad (4)$$

By calculating the above integral, we find

$$I_{MAI,i,k} = \frac{1}{2} c_{h,i}(t) c_{k,i}(t) \cos\theta_{k,i} \cdot \left[\alpha_{k,i,-1} b_{k,-1} \tau_k + \alpha_{k,i,0} b_{k,0} (T_s - \tau_k)\right] \qquad (5)$$

where $\theta_{k,i} = 2\pi f \tau_k - \psi_{k,i}$ is a phase offset, $\alpha_{k,i,-1}$ and $\alpha_{k,i,0}$ are the channel Rayleigh fading parameter affecting the previous and current data bit respectively.

The total MAI affecting the reference user is the sum of the MAI contributions from each user and each sub-carrier:

$$I_{MAI,tot} = \sum_{\substack{k=1 \\ k \neq h}}^{K} \sum_{i=1}^{N} I_{MAI,i,k} \qquad (6)$$

3.2 Inter-Channel Interference

The ICI introduced by the sub-carriers of the kth user into the i^{th} subcarrier of the reference user h over one symbol duration is given by

$$I_{ICI,k,i} = \sum_{\substack{j=1 \\ j \neq i}}^{N} I_{ICI,k,i,j} \qquad (7)$$

where $I_{ICI,i,k}$ represents the ICI introduced by the j^{th} subcarrier of user k into the i^{th} subcarrier of the reference user. It is given by [2]

$$I_{ICI,i,j,k} = \int_0^{T_s} \alpha_{k,j} b_k(t-\tau_k) \cdot c_{h,i}(t) c_{k,j}(t-\tau_k) \cos[2\pi f_i t] \cdot \cos[2\pi f_j (t-\tau_k) + \psi_{k,i}] \tag{8}$$

The above integral can be expressed as

$$I_{MAIDF,i,j,k} = \frac{1}{2}\left[R_{i,j}(\tau_k) + \hat{R}_{i,j}(\tau_k)\right] \tag{9}$$

with

$$R_{i,j}(\tau_k) = \int_0^{\tau_k} \alpha_{k,j,-1} b_{k,-1} c_{k,j}(t) c_{h,j}(t) \cdot \cos\left[2\pi(f_i - f_j)t + \theta_{k,j}\right] dt \tag{10}$$

$$R_{i,j}(\tau_k) = \int_{\tau_k}^{T_s} \alpha_{k,j,0} b_{k,0} c_{k,j}(t) c_{h,j}(t) \cdot \cos\left[2\pi(f_i - f_j)t + \theta_{k,j}\right] dt \tag{11}$$

After evaluation of the integral, the total ICI for the i^{th} subcarrier due to user k is obtained as

$$I_{ICI,i,k} = \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{T_s c_{k,j}(t) c_{h,i}(t)}{4\pi(i-j)} \cdot \left(\alpha_{k,j,-1} b_{k,-1} - \alpha_{k,j,0} b_{k,0}\right) \cdot \left[\sin\left(\frac{2\pi(i-j)}{T_s}\tau_k + \theta_{k,j}\right) - \sin\theta_{k,j}\right] \tag{12}$$

The total ICI affecting the reference user is the sum of the ICI contributions from each user, i.e.

$$I_{ICI,tot} = \sum_{\substack{k=1 \\ k\neq h}}^{K} \sum_{i=1}^{N} I_{ICI,i,k} \tag{13}$$

4. 2nd Moment of Interference

The total interference of asynchronous MC-CDMA system is generally assumed Gaussian distributed with zero mean [2]. In order to obtain the expression of second moment of MAI in terms of the timing offset, τ_k, for user k, we take the expectation with respect to $b(t)$, α and θ conditionally to τ_k.

$$\begin{aligned} I^2_{MAI,tot}(\tau_k) &= E_{b(t),\alpha,\theta}[\sum_{\substack{k=1 \\ k\neq h}}^{K} I^2_{MAI,k} \mid \tau_k] \\ &= E_{b(t),\alpha,\theta}[\sum_{\substack{k=1 \\ k\neq h}}^{K} \sum_{i=1}^{N} I^2_{MAI,i,k} \mid \tau_k] \\ &= \sum_{\substack{k=1 \\ k\neq h}}^{K} \sum_{i=1}^{N} E_{b(t),\alpha,\theta}[I^2_{MAI,i,k} \mid \tau_k] \\ &= \frac{\beta^2}{4} \sum_{\substack{k=1 \\ k\neq h}}^{K} \sum_{i=1}^{N} [\tau_k^2 + (T_s - \tau_k)^2] \end{aligned} \tag{14}$$

where $E_{b(t),\alpha,\theta}[\ \mid\tau_k]$ represents the expected value with respect to b(t), α and θ conditionally to τ_k. Similarly, the ICI power for reference user h can be written as

$$I^2_{ICI,tot}(\tau_k) = \frac{T_s^2 \beta^2}{4\pi^2} \sum_{\substack{k=1 \\ k\neq h}}^{K} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{1}{(i-j)^2} \cdot \left[1 - \cos\left(\frac{2\pi(i-j)}{T_s}\tau_k\right)\right] \tag{15}$$

Hence the total interference power in terms of τ is given as

$$I^2_{h,tot}(\tau_k) = I^2_{MAI,tot}(\tau_k) + I^2_{ICI,tot}(\tau_k) \tag{16}$$

4.1 Case 1: τ_k is uniformly distributed over [0 T_s]

By taking the expectation of (12) and (13) further respect to τ_k, the 2nd moment of MAI and ICI power are obtained as

$$\begin{aligned} E\left[I^2_{MAI,tot}\right] &= (K-1)\sum_{i=1}^{N} E_{\tau_k}\left[I^2_{MAI,i,k}(\tau_k)\right] \\ &= (K-1)\left(\frac{\beta^2 N T_s^2}{6}\right) \end{aligned} \tag{17}$$

$$\begin{aligned} E\left[I^2_{ICI,tot}\right] &= (K-1)\sum_{i=1}^{N} E_{\tau_k}\left[I^2_{ICI,i,k}(\tau_k)\right] \\ &= \frac{(K-1)\beta^2 T_s^2}{2\pi^2}\left(\sum_{l=1}^{N-1}\frac{N}{l^2} - \sum_{l=1}^{N-1}\frac{1}{l}\right) \end{aligned} \tag{18}$$

Therefore the 2nd moment of the total interference power is

$$E[I^2_{h,tot}] = E[I^2_{MAI,tot}] + E[I^2_{ICI,tot}] \tag{19}$$

Similar expression has previously been obtained by Hanzo in [2] using a similar approach that was, however, restricted to uniformly distributed offsets.

4.2 Case 2: τ_k is Poisson distributed

The conditional probability density function (pdf) for Poisson distribution is given as

$$f(x/\lambda)=e^{-\lambda}\frac{\lambda^{x}}{x!} \qquad (20)$$

Where λ is the Poisson distribution parameter and is assumed to be the same for all channels. λ also represents the mean and variance of the distribution and we restrict it, in this paper, to the interval [0 T_s). Hence, the timing offsets lie, on average, within [0 T_s), but can, in theory, take any value between '0' to '∞'. As a result, interference does not only affect the current and previous information bits as in the uniform distributed case, but can also affect all the preceding bits. Our simulation results show, however, that as long as λ is within the range of [0 T_s), we can achieve a good approximation for the total interference power by only considering the current bit 'b_0', and the previous two bits *'b_{-1}'* and *'b_{-2}'*. Further, the closer λ is to 0, the less significant the timing offsets outside [0 $2T_s$) and the smaller the approximation error. These results are displayed in *Table 1*, in the next section. We now derive the interference power in the Poisson case, by restricting the interference to the two previous bits.

The interval [0 T_s) is first sampled using M samples. The need for sampling arises from the need for using integer values in the Poisson distribution. The conditional 2nd order moment of the MAI for τ_k in the range of [0, Ts] can then be written as

$$E[I^2_{MAI,tot,\tau_k=[0,T_s]}]=\sum_{\substack{k=1\\k\neq h}}^{K}E_{\tau_k}[I^2_{MAI,tot,\tau_k=[0,T_s]}(\tau_k/\lambda)]$$

$$=\frac{(K-1)T_s^2\beta^2N}{4M^2}E_{\tau_k}\left[\tau_k{}^2+(M-\tau_k)^2\right] \qquad (21)$$

$$=\frac{(K-1)T_s^2\beta^2Ne^{-\lambda}}{4M^2}\sum_{\tau_k=1}^{M}\left[\tau_k{}^2+(M-\tau_k)^2\right]\frac{\lambda^{\tau_k}}{\tau_k!} \qquad (22)$$

For τ_k in the range of (T_s $2T_s$), we can simply replace τ_k in (19) with (τ - M), which gives

$$E[I^2_{MAI,tot,\tau_k=(T_s,2T_s)}]$$
$$=\frac{(K-1)T_s^2\beta^2Ne^{-\lambda}}{4M^2} \qquad (23)$$
$$\cdot\sum_{\tau_k=M+1}^{2M}\left[(\tau_k-M)^2+(2M-\tau_k)^2\right]\frac{\lambda^{\tau_k}}{\tau_k!}$$

The conditional 2nd order moment of the ICI can be derived in a similar way. For τ_k in the range of [0, Ts], we can develop the expression of the 2nd moment ICI as follows

$$E[I^2_{ICI,tot,\tau_k=[0,Ts]}]=\sum_{\substack{k=1\\k\neq h}}^{K}E_{\tau_k}[I^2_{ICI,tot,\tau_k=[0,T_s]}(\tau_k/\lambda)]$$

$$=\frac{(K-1)T_s^2\beta^2}{4\pi^2}E_{\tau_k}\left[\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{(i-j)^2}[1-\cos(\frac{2\pi(i-j)}{M}\tau_k)]\right] \qquad (22)$$

$$=\frac{(K-1)T_s^2\beta^2e^{-\lambda}}{4\pi^2}\sum_{\tau_k=1}^{2M}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{(i-j)^2}[1-\cos(\frac{2\pi(i-j)}{M}\tau_k)]\cdot\frac{\lambda^{\tau_k}}{\tau_k!}$$

(24)

For the case when τ in the range of (Ts, 2Ts), we again replace τ_k in (22) with (τ_k - M).

$$E[I^2_{ICI,tot,\tau_k=[Ts,2Ts]}]=\sum_{\substack{k=1\\k\neq h}}^{K}E_{\tau_k}[I^2_{MAI,tot,\tau_k=[Ts,2Ts]}(\tau_k/\lambda)]$$

$$=\frac{(K-1)T_s^2\beta^2e^{-\lambda}}{4\pi^2}\sum_{\tau_k=M+1}^{2M}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{(i-j)^2}[1-\cos(\frac{2\pi(i-j)}{M}(\tau_k-M))]\cdot\frac{\lambda^{\tau_k}}{\tau_k!}$$

$$=\frac{(K-1)T_s^2\beta^2e^{-\lambda}}{4\pi^2}\sum_{\tau_k=M+1}^{2M}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{(i-j)^2}[1-\cos(\frac{2\pi(i-j)}{M}\tau_k)]\cdot\frac{\lambda^{\tau_k}}{\tau_k!}$$

(25)

By comparing (23) and (24), we find that these two formulas are effectively the same. Hence, we can combine (23) and (24), which gives,

$$E[I^2_{ICI,tot,\tau_k=[Ts,2Ts]}]$$

$$=\frac{(K-1)T_s^2\beta^2e^{-\lambda}}{4\pi^2}\sum_{\tau_k=1}^{2M}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{(i-j)^2}[1-\cos(\frac{2\pi(i-j)}{M}(\tau_k-M))]\cdot\frac{\lambda^{\tau_k}}{\tau_k!}$$

(26)

We then obtain the 2nd order moment of the total interference by summing up equations (21), (22) and (25) which gives

$$E[I^2_{h,tot}]$$
$$=E[I^2_{MAI,tot,\tau_k=[0,T_s]}]+E[I^2_{MAI,tot,\tau_k=[T_s,2T_s]}]+E[I^2_{ICI,tot}]$$

$$= \frac{(K-1)T_s^2\beta^2 e^{-\lambda}}{4} \times$$

$$\left[\begin{array}{l} \frac{N}{M^2}\sum_{\tau_k=1}^{M}\left[\tau_k^{\,2}+(M-\tau_k)^2\right]\frac{\lambda^{\tau_k}}{\tau_k\,!} \\ +\frac{N}{M^2}\sum_{\tau_k=M+1}^{2M}\left[(\tau_k-M)^2+(2M-\tau_k)^2\right]\frac{\lambda^{\tau_k}}{\tau_k\,!} \\ +\frac{1}{\pi^2}\sum_{\tau_k=1}^{2M}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{1}{(i-j)^2}[1-\cos(\frac{2\pi(i-j)}{M}\tau_k]\cdot\frac{\lambda^{\tau_k}}{\tau_k\,!} \end{array} \right]$$

(27)

5. Statistical Model of Asynchronous MC-CDMA

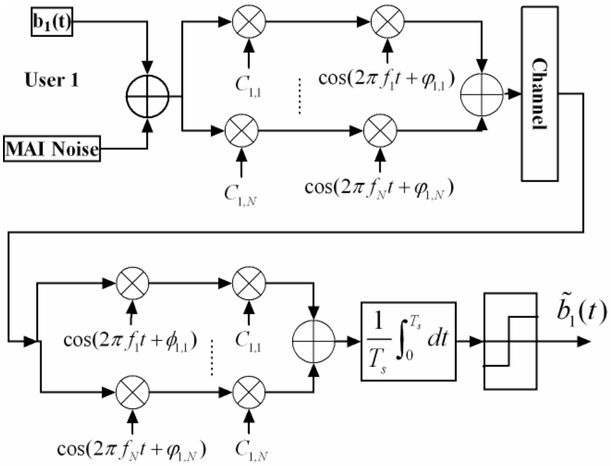

Figure2. Proposed Asynchronous MC-CDMA Model

With the knowledge of the 1st and 2nd order moments of the total interference, we are able to propose a computer simulation model that can significantly reduce the computational load when simulating asynchronous MC-CDMA systems. The proposed model is shown in *Figure 2.* It is based on replacing the asynchronous model with a synchronous model followed by an additive noise representing the total interference affecting asynchronous MC-CDMA systems. This allows for a significant reduction of computational load. Indeed, in order to simulate the asynchronous model and evaluate its performance in a given channel, signals from all interfering users with different randomly generated offsets need to be simulated for each reference user. In the proposed model, only reference users are simulated. The requirement for simulation of interfering users is replaced by adding a noise component representing the interference. The noise is zero-mean, white, Gaussian, with a power that has been derived in the previous section.

In next section, the proposed model is validated using Monte-Carlo simulations, by comparing the BER at the output of the asynchronous MC-CDMA system to that at the output of the model. The results are also shown in Table 2.

6. Simulation Results

An asynchronous MC-CDMA system with 8 users using an 8-Length Walsh-Hadamard (WH) Code is simulated. The Rayleigh parameter for the channel is assumed equal to 1. The number of realizations used for the Monte-Carlo simulations is 250000.

6.1 Interference Power

We first validate the results of equations (19) and (27) by computing the interference power in the asynchronous MC-CDMA system using Monte-Carlo simulations for both uniformly distributed and Poisson distributed timing offsets. For Poisson distributed timing offsets, we also computed simulations for different values of the Poisson parameter, i.e. λ =0.1 T_s , λ =0.5 T_s and λ =0.9 T_s. The results are displayed below.

Table 1 Comparison of 2nd Moment of Interference

Uniform Distribution	Simulation	Formula (17)	Error	Poisson Parameter
	12.841	12.7029	1.09%	N.A.
Poisson Distribution	Simulation	Formula (27)	Error	
	12.8582	12.8898	0.25%	0.1 T_s
	12.4781	12.4689	0.07%	0.5 T_s
	12.9277	12.817	0.86%	0.9 T_s

It can be seen that equations (19) and (27) provide a very good representation of the interference power in asynchronous MC-CDMA systems for uniformly distributed and Poisson distributed timing offsets respectively.

6.2 Comparison of BER between two models

Using the same simulation conditions described above, the BER performance between the asynchronous MC-CDMA and the proposed computer simulation model is compared. The results are shown in *Table 2*. It can be seen that for all different distributions of timing offset τ, the BER error is below 3%.

Although computational time can be different depending on the computer hardware used, we claim that the new model uses approximate one tenth of the computational time required to compute the conventional asynchronous MC-CDMA system.

Table 2 – BER performance comparison

	Conventional Model	Proposed Model	Error	Poisson Parameter
Normal Distribution	0.1342	0.1303	2.99%	NA
Poisson Distribution	0.1284	0.1286	0.16%	0.1 T_s
	0.1262	0.1265	0.24%	0.5 T_s
	0.1344	0.1308	2.75%	0.9 T_s

7. Conclusion

In this paper, we derived the expression of the conditional power of the MAI and ICI in asynchronous MC-CDMA systems. This expression is given as a function of timing offset τ. Using this result, we derived the 2^{nd} moment of the total interference under two conditions 1) τ is assumed to be uniformly distributed over [0, Ts); 2) τ is assumed to be Poisson distributed with its parameter λ ranging within [0, Ts). With the knowledge of the 1^{st} and 2^{nd} order moment of the total interference, a computer simulation model is proposed which is proved to have comparable BER performance to the traditional asynchronous MC-CDMA system, but requires much less computational time.

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